

Changes in Mechanical Properties in Metal-Forming Processes

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This article presents a paradigm for the mechanical properties of a metal that result from the plastic deformation that occurs in forming processes, including multiple-step or cyclic processes. Some of the new concepts included are a new strength designation that includes the sense (tensile or compressive), the direction (longitudinal, transverse, etc.), the specific type of deformation (tensile, compressive, etc.); a concept of an equivalent strain; and the apparent rules of strain strengthening. Examples of strength analyses are also included.

Keywords
equivalent strain, plastic deformation, strain strengthening, strength

1. Introduction

THE tedious and time-consuming work of the stress analyst of the past century is rapidly being replaced by very efficient, user friendly finite-element modeling software programs. The computer displays of these programs show not only the magnitude of the stress at any point in the part being analyzed, but they also indicate the sense (tensile or compressive) of the stress and the direction in which it acts.

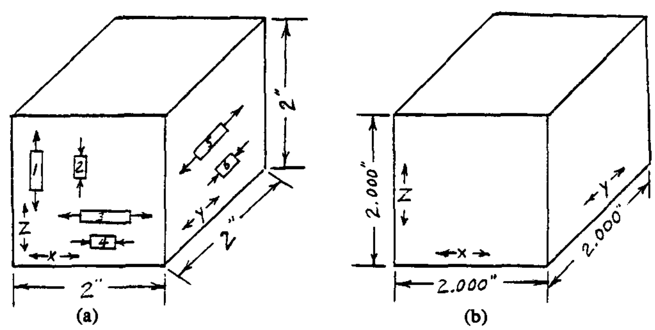
Unfortunately, the mechanical reliability of the fabricated components is not solely dependent on the accuracy of the stress analysis. Of equal importance is the accuracy of the determination of the strength of the material at the locations of the calculated stresses. In other words, the reliability of the part depends on the accuracy of the strength-to-stress ratio. Although the stress analysis may be accurate to six significant figures, the strength determination may be very unreliable. In fact, it is possible for the value listed in some materials handbooks for a given alloy to be in error by 100%. The author has in his files several examples of errors of this magnitude.

One of the main reasons for the presence of such large errors in published values of strength is the poor documentation of the particular strength in question. Whereas the stress is specified with respect to magnitude, sense, and direction, the strength is specified only in terms of its magnitude. Although this may be sufficient for noncold worked metals, it is not satisfactory for plastically deformed metals. A metal that has been cold worked has values of strength that are different in the sense of the strength (tensile or compressive), the direction within the part, and the sense of the deforming strain during the fabrication of the part.

Formed parts are being used more extensively each year in the manufacture of automobiles, airplanes, home appliances, and all other mechanical devices. To design such parts so that they have a high mechanical reliability along with minimum cost or weight, it is necessary for the engineer to know the sense and direction of the strength in the part in addition to its magnitude.

Figure 1 is an example of the need for a strength designation that includes the sense of the strength, the direction of the strength, and the type of previous plastic deformation. This figure summarizes one of hundreds of tests on a variety of ferrous and nonferrous metals and a variety of deformation processes performed by the author and his students.

Figure 1(a) represents a 2-in. cube of annealed brass (No. 260), from which three tensile and three compression speci-



Spec. No.	Yield strength (MPa) Tens.	Compr.
1	72.8	----
2	----	74.5
3	76.5	----
4	----	80.7
5	77.9	----
6	----	77.9
7	170	----
8	----	316
9	272	----
10	----	245
11	263	----
12	----	239

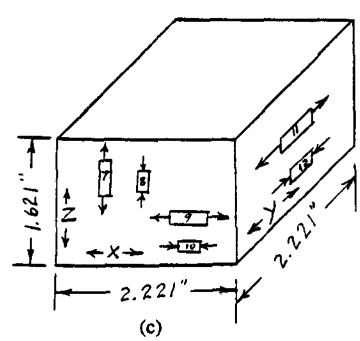


Fig. 1 Directional strength in a cold worked part. **(a)** Tensile and compressive tests on an annealed block of alloy No. 260 brass. **(b)** Second block identical to (a). **(c)** Block (b) after 19% cold work by compression in the Z-direction.

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mens were tested, one in each of the three principal directions. The tensile and compressive yield strengths of all six were approximately the same—slightly more than 76 MPa (11 ksi).

Figure 1(b) represents an identical annealed cube that was subsequently compressed (19% cold work) in the Z-direction to the size shown in Fig. 1(c). Three tensile and three compression specimens were machined from the deformed block in the same directions as those from the annealed cube. Note the great range of values for the 0.2% offset yield strengths. The compressive yield strength in the Z-direction is nearly 100% greater than the value of the tensile yield in that direction, and the yield strengths in the transverse direction are intermediate to those in the longitudinal direction. This variation in strength is not an exception. It occurs with all metals to some degree. It is most severe with many stainless steels, copper alloys, cobalt alloys, etc. Even the ordinary steels exhibit a significant variation.

The materials handbooks list only one value of yield strength for each amount of cold work. For example, the Revere Copper and Brass booklet lists the yield strength of alloy No. 260 as 345 MPa, with 20% cold work. It does not document whether the value is for the tensile or the compressive yield, nor does it indicate the orientation within the part, or the type of deformation to which the material was subjected. Designs based on such poorly specified data can certainly produce unreliable parts that result in unexplained failures. As Fig. 1 shows, the actual tensile yield strength in one direction can be as low as 50% of the value listed in the handbooks.

2. A New Strength Designation

To avoid this great confusion and unreliability of mechanical property data, it is necessary to have a clear and precise method of designating the strength of a plastically deformed metal. The author created[1] such a designation in 1966. During the ensuing years, he has found it to be extremely beneficial and reliable. It is now included in the new *Standard Handbook of Mechanical Design* published by McGraw Hill.

The symbol used to denote strength is the capital letter S. This distinguishes it from stress, for which the Greek letter sigma (σ) is commonly used. The S must be followed by four subscripts to identify the (1) type of strength, (2) sense of the strength, (3) direction of the strength, and (4) sense of the last plastic deformation strain. It is extremely beneficial to the readability of this notation to place parenthesis in front of the letter S and after the first subscript. A typical example of this notation is $(S_u)_{tLc}$, which represents the ultimate tensile strength in the longitudinal direction after prior compression in that direction.

The first subscript specifies the type of strength under investigation, which may be any of the following: y, for yield; u, for ultimate; f, for fracture; e, for endurance limit; and p, for proportional limit.

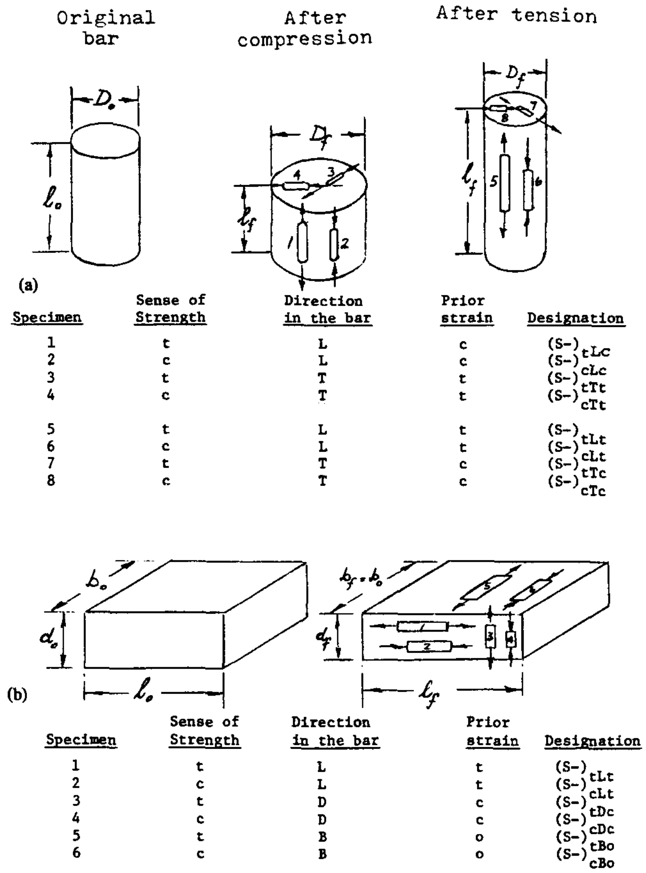
The second subscript refers to the sense of the strength, which may be any of the following: t, for tensile; c, for compression; and s, for shear.

The third subscript indicates the direction or orientation of the test specimen within the formed part. For clarity, capital letters are used for the third subscript. It may be an L for a specimen whose axis is parallel to the direction of the principal deforming strain. For symmetrical cross sections such as rounds or squares, it may be a T for transverse; an R for radial; or a C for circumferential. These latter three are all equivalent. In the case of plates or rectangular bars, two transverse directions must be specified: the long transverse can be indicated by a B (breadth); the short transverse can be designated with an H (thickness).

The fourth subscript refers to the sense of the last prior strain in the axial direction of the test specimen, but not in the direction of the deforming force. The notation here is the same as for the second subscript; that is, t, for tension; c, for compression; or s, for shear. However, if there was no prior strain, then the subscript o should be used. For a non-cold-worked or heat treated isotropic material, the second, third, and fourth subscripts can be deleted.

To generalize, the strength of a cold worked material can be represented as $(S\text{-})_w$ and that of the original non-cold-worked material as $(S\text{-})_o$.

A second example is given to illustrate this designation. Consider the notation $(S_y)_{cTc}$, which means the compressive yield strength in the transverse direction after compressive strain in the transverse direction (equivalent to an extension in the longitudinal direction).



(a)

Specimen	Sense of Strength	Direction in the bar	Prior strain	Designation
1	t	L	c	$(S\text{-})_{tLc}$
2	c	L	c	$(S\text{-})_{cLc}$
3	t	T	t	$(S\text{-})_{tTt}$
4	c	T	t	$(S\text{-})_{cTt}$
5	t	L	t	$(S\text{-})_{tLt}$
6	c	L	t	$(S\text{-})_{cLt}$
7	t	T	c	$(S\text{-})_{tTc}$
8	c	T	c	$(S\text{-})_{cTc}$

(b)

Specimen	Sense of Strength	Direction in the bar	Prior strain	Designation
1	t	L	t	$(S\text{-})_{tLt}$
2	c	L	t	$(S\text{-})_{cLt}$
3	t	D	c	$(S\text{-})_{tDc}$
4	c	D	c	$(S\text{-})_{cDc}$
5	t	B	o	$(S\text{-})_{tBo}$
6	c	B	o	$(S\text{-})_{cBo}$

Fig. 2 Strength designation after **(a)** uniaxial deformation and **(b)** rolling of a plate.

Four strength designations, as illustrated in Fig. 2, are needed to completely specify the strength in bar or plate that is upset (compressed in the longitudinal direction). The tensile and compressive strengths in the longitudinal direction are denoted as $(S\text{-})_{tLc}$ and $(S\text{-})_{cLc}$, respectively. The dash can be replaced by any of the symbols discussed above that can be used for the first subscript. The tensile and compressive strengths in the transverse direction are expressed as $(S\text{-})_{tTt}$ and $(S\text{-})_{cTt}$, respectively. In this case, the fourth subscript t indicates that the prior strain in the transverse direction was tensile during the compressive deformation.

Four strength designations are also needed for a bar that has been axially (longitudinally) stretched. In the longitudinal direction, the tensile and compressive strengths are $(S\text{-})_{tLt}$ and $(S\text{-})_{cLt}$, respectively. Similarly, in the transverse direction, they are $(S\text{-})_{tTc}$ and $(S\text{-})_{cTc}$. Again, the last subscript c indicates that the prior strain in the transverse direction was compressive.

In the case of rolling or bending of a plate (width >8 × thickness), six designations are necessary. There are both tensile and compressive strengths in each of the three perpendicular directions: longitudinal, L; long transverse, B; and short transverse, H. Thus, the six designations are $(S\text{-})_{tLt}$ and $(S\text{-})_{cLt}$; $(S\text{-})_{tHc}$ and $(S\text{-})_{cHc}$; $(S\text{-})_{tBo}$, and $(S\text{-})_{cBo}$. The last subscripts are t, for the longitudinal direction because the length increases; c, for the short transverse because the thickness is reduced; and o, for the long transverse because the width remains constant.

When the desired strength is at some angle other than 0° or 90°, that is other than longitudinal or transverse, then a simple linear interpolation of the strength on the basis of the 0° and 90° values is required. Thus, if a flat bar is uniaxially deformed in tension and if the tensile yield strength at an angle of 30° to the longitudinal direction is desired, then the strength is calculated as follows:

$$(S_y)_t = 2/3(S_y)_{tLt} + 1/3(S_y)_{tTc}$$

Likewise, the compressive yield strength is calculated as follows:

$$(S_y)_c = 2/3(S_y)_{cLt} + 1/3(S_y)_{cTc}$$

The techniques for calculating the numerical values for these strengths are presented in the following section.

3. The Equivalent Strain

To calculate the strength of a metal that has been subjected to one or more cycles of plastic deformation, it is necessary to know the value of the equivalent strain. In this context, the equivalent strain is the value of the accumulated strain during the deformation processes, which when used in conjunction with the monotonic plastic tensile stress-strain properties enables the multidirectional strength of the material to be calculated.

Borden[2] developed a reliable model of the equivalent strain ε_{qu}, which is defined by the equation:

$$\varepsilon_{qu} = \sum_{i=1}^{n} \frac{\varepsilon_i}{a+1}$$

where i is the order number of a strain cycle when the strains are listed in order of decreasing magnitude; ε_i is the maximum value of strain during cycle No. i, disregarding the sense (– or +) or direction of the strain; n is the total number of cycles of strain; $a = 0$ for the strengths that have the same sense as the last prestrain. That is, when the subscripts 2 and 4 in the strength designation have the same sense, or when the 4th subscript is zero; and $a = 1$ for strengths having the opposite sense at the last prestrain. That is, when the subscripts 2 and 4 in the strength designation are of the opposite sense.

The value of ε_w for any one deformation step that should be used to calculate ε_{qu} is the largest of the three principal strains. A cycle of strain occurs whenever there is a reversal or change of the sense of the strain in any one of the three principal directions.

4. The Apparent Rules of Strain Strengthening

The term "apparent rules" is used because they portray what the strength of a cold worked material appears to be on the basis of a broad range of experimental studies. They have been developed by the author, with the help of students, during the past 25 years, while attempting to present to engineering students, in a systematic manner, the results of many individual research projects in the fields of plastic deformation and mechanical properties. Included among these projects are cyclic axial deformation of cylinders, cyclic deformation of large cubes in the three perpendicular directions, bending and unbending of flat specimens, cyclic torsional deformation of cylinders, shearing of blanks and strips, deep drawing of channel sections and cylindrical cups, wire drawing, forward and back extrusion, and cold rolling. A variety of tensile, compressive, fatigue, and hardness tests were conducted on the cold worked metals.

The strength in any direction of a cold worked part is calculated by integrating the equivalent strain into the plastic monotonic tensile stress-strain equation. For purposes of clarity and preciseness in presenting this subject to engineering students, the exponential strain-strengthening equation is expressed as:

$$\sigma = \sigma_o \varepsilon^m$$

where m is the strain strengthening exponent, and σ_o is the stress coefficient.

The values of strength calculated by means of the following rules agree with experimentally determined values in most cases to an accuracy of less than 5% and in a few cases to less than 10%. This is true for metals that have been subjected to as many as five cycles of strain and where the yield strength has been increased from a value of 8 ksi prior to cold work to 524 MPa after cold work. The calculations made on the basis of these rules show the effect of cold work alone on the strength of the material. They are valid when no other strengthening mechanism such as precipitation or recrystallization occurs after the cold working process.

4.1 *Rule 1*

Strain strengthening is a bulk mechanism. Even a deformation load that is applied in only one direction causes deformation and strengthening in all directions. For example, in the cold rolling of a plate, even though there is no increase in the breadth of the plate, both the tensile and compressive yield strengths in the breadth direction are increased by 10 to 100%, depending on the amount of cold work.

4.2 *Rule 2*

The maximum deformation that can be given a material during the forming of a part is the deformation that induces a tensile strain that is numerically equal to the fracture strain of that material when tested in the same direction and identical environment. In other words, fracture will occur when the induced tensile strain ε_t is equal to the fracture strain ε_f. This has previously been referred to by the author as the Failure Theory for Plastic Working.[3] However, from a practical approach, a few forming operations are terminated when the induced tensile strain is equal to the ultimate load strain, which is also numerically equal to the strain-strengthening exponent. This is true for those forming operations in which the condition known as necking occurs.

Some operations are a combination of stretching and bending, and in these cases, the failure occurs at some value of strain between ε_u and ε_f, depending on the ratio of bending to stretching strain.

4.3 *Rule 3*

The numerical value of ε_w in any one deformation cycle to be used in calculating the equivalent strain ε_{qu} is the largest of the three strains present at any location. It is usually the longitudinal or axial strain. For example, if a cube is compressed in the *Z*-direction to a strain of –0.2, the strains in the *X* and *Y* directions are each +0.1. The numerical value of strain that is used for ε_w in this case is 0.2, even when calculating the strength in the transverse direction.

4.4 *Rule 4*

In determining the value of ε_{qu}, the values of ε_{wi} are added in order of decreasing numerical value rather than in their chronological order, as defined in Table 1, and without regard to their sign. Thus, for a strain sequence of +0.10, –0.20, +0.15, –0.05, the value of ε_{qus} (ε_{qu} for $a = 0$) is 0.20 + 0.15/2 + 0.10/3 + 0.05/4 or 0.32. The value of ε_{quo} (ε_{qu} for $a = 1$) is 0.20/2 + 0.15/3 + 0.10/4 + 0.05/5 or 0.19. This method of adding the strains gives the greatest effect to the largest strain regardless of when it occurred in the actual strain sequence.

For only one cycle of strain, $\varepsilon_{qus} = \varepsilon_w$ and $\varepsilon_{quo} = 0.5\ \varepsilon_w$.

As stated previously when the concept of the equivalent strain was presented, the strength of the cold worked metal is calculated by combining the equivalent strain with the plastic stress-strain relationship of the metal, as explained in rules 5 and 6.

4.5 *Rule 5*

The tensile strength of a cold worked metal is determined by means of the two equations:

$$(S_u)_w = (S_u)_o e^{\varepsilon_{qu}} \text{ for } \varepsilon_{qu} \le m$$

$$(S_u)_w = \sigma_w = \sigma_o(\varepsilon_{qu})^m \text{ for } \varepsilon_{qu} \ge m$$

Table 1 Equations for calculating yield strength

Group No.	Yield strengths	Equation
1	$(S_y)_{tLt}$ $(S_y)_{cLc}$ $(S_y)_{tBo}$ $(S_y)_{cBo}$ $(S_y)_{cHc}$	$= \sigma_o\left(\dfrac{\varepsilon_{qus}}{1 + 0.2\varepsilon_{qus}}\right)^m$
2	$(S_y)_{tTt}$ $(S_y)_{cTc}$	$= \sigma_o\left(\dfrac{\varepsilon_{qus}}{1 + 0.5\varepsilon_{qus}}\right)^m$
3	$(S_y)_{tTc}$ $(S_y)_{cTt}$	$= 0.90(S_y)_{tTt}$
4	$(S_y)_{cLt}$ $(S_y)_{tLc}$ $(S_y)_{tHc}$	$= \sigma_o\left(\dfrac{\varepsilon_{quo}}{1 + 2\varepsilon_{quo}}\right)^m$

The value of ε_{qu} that should be used in the above two equations is ε_{qus} ($a = 0$) if the last deforming strain in the direction of the desired strength was tensile. If the last deforming strain in the direction of the desired strength was compressive, then ε_{quo} ($a = 1$) must be used.

4.6 *Rule 6*

The yield strength of a cold worked metal is determined by means of the equation:

$$(S_y)_w = \sigma_o(\varepsilon_{qy})^m$$

where ε_{qy} is the equivalent yield strain as defined in Table 1.

The yield strengths are divided into four groups as shown in Table 1. For any given strain sequence, the numerical values of all the yield strengths in any one group are equal. The largest strengths are in Group 1, and the smallest are in Group 4.

5. Examples of Strength Analysis

The application of the above concepts and relationships can best be illustrated by considering three typical strength analysis problems. Consider first the problem in which a 2.5-in. diameter head is cold upset on the end of a 2-in. diameter shaft prior to machining splines on the enlarged end. Also, to save some machining, a cold drawn bar rather than an annealed one is selected. Assume that in the cold drawing operation a 2.25-in. diameter bar was reduced to the 2-in. diameter. The material selected is a type 304 stainless steel with a yield strength of 341 MPa, a tensile strength of 593 MPa, $\sigma_o = 1379$ MPa, $m = 0.50$, and $\varepsilon_f = 1.7$.

Because the force acting on the splines is in the circumferential, or transverse, direction it is necessary to know both the tensile and compressive strengths in that direction. The first cold drawing strain is calculated as:

$$\varepsilon_1 = 2 \ln 2.25/2.00 = 0.24$$

The upsetting strain is:

$$\varepsilon_1 = 2 \ln 2.00/2.50 = -0.45$$

According to the notations and equations in Table 1, the equivalent strains are calculated as:

$$\varepsilon_{qus} = 0.45 + 0.24/2 = 0.57$$

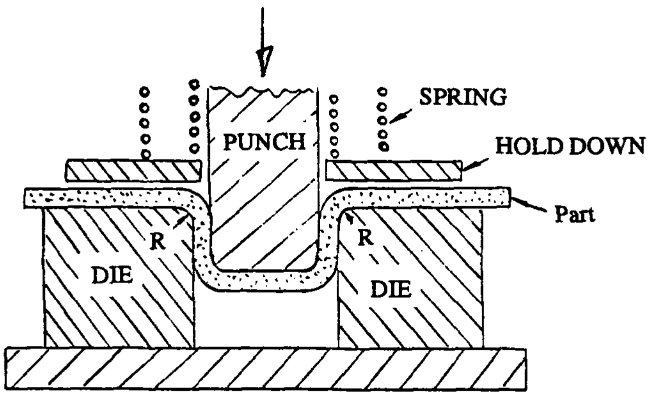


Fig. 3 Draw die to a hat (channel) section.

$\varepsilon_{quo} = 0.24/2 + 2/24/3 = 0.30$

The transverse tensile yield strength is calculated as:

$(S_y)_{tTt} = \sigma_o(\varepsilon_{qys})^m$

where

$\varepsilon_{qys} = 0.57(1 + 0.5 \times 0.57) = 0.44$

$(S_y)_{tTt} = 1379(0.44)^{0.5} = 917$ MPa

The transverse compressive yield strength is calculated as:

$(S_y)_{cTt} = 0.90((S_y)_{tTt}) = 0.90 \times 917 = 827$ MPa

However, if the properties were measured experimentally in the longitudinal direction, they would more closely resemble the following calculated values:

$(S_y)_{cLc} = 1379[0.57/(1 + 0.2 \times 0.57)]^{0.5} = 986$ MPa

The tensile strength in the transverse direction is:

$(S_u)_{tTt} = 1379(0.57)^{0.5} = 1041$ MPa

In the longitudinal direction, the tensile strength is:

$(S_u)_{tLc} = 593^{0.30} = 800$ MPa

Next, consider a slightly more complicated strength analysis problem. A channel section is made by deep drawing 1/8-in. thick by 6-in. wide by 30-in. long strips over a 1/8-in. die radius (see Fig. 3). The finished channel has 2-in. high legs. The original material is 3003-0 aluminum having the following properties:$(S_y)_o = 41$ MPa; $(S_y)_o = 103$ MPa; $\sigma_o = 200$ MPa; $m = 0.30$; $\varepsilon_f = 1.50$.

The question is, "What are the numerical values of the tensile yield strength at the outside surface and at the midthickness of the drawn legs in the direction of the 30-in. length?"

The calculations are based on the fact that the neutral axis remains at the midthickness during pure bending. With the proper hold-down pressure and good lubrication, thinning of the metal during bending and unbending is negligible, and therefore, the inner fiber compressive strain is considerably greater than the outer fiber tensile strain.

The strain at any distance Y from the neutral axis is calculated from the expression $\varepsilon = \ln [(R + Y)/(R + N)]$, where R is the inside radius and N is the distance the neutral axis lies above R. In this case, $N = h/2$.

By referring to Fig. 3, it is apparent that the outside surface of the leg is the inner fiber during the first bending of the leg. Therefore, the bending strain $\varepsilon_1 = \ln [(0.125 + 0)/(0.125 + 0.0625)] = -0.405$. As the drawing operation continues, the bent metal is unbent with an equal tensile strain of 0.405 because the finished leg is straight.

The tensile yield strength in the long transverse direction during bending is designated $(S_y)_{tBo}$ because there is no strain in the 30-in. direction. The value of ε_{qus} is 0.405 + 0.405/2 = 0.608. The tensile yield strength at the outside surface in the 30-in. direction can be calculated as follows:

$(S_y)_{tBo} = 200\,[0.608/(1 + 0.2 \times 0.608)]^{0.30} = 166$ MPa. This is an increase of 300% over the original yield strength.

The calculation of the yield strength at the midthickness is slightly more complicated. Although the neutral axis remains at the midthickness, some metal originally below the midthickness is displaced to a position above the midthickness. For example, the metal that ends up at the midthickness (0.0625-in.) was originally 0.0521-in. above the bottom surface of the flat strip. Also, when the instantaneous radius is 0.310-in., this particular element is subjected to a compressive strain of –0.014. Then as the radius is further reduced to 0.125-in., a tensile strain of 0.014 is induced in this element, which is now at the midthickness with a final net strain of 0. All of this occurs during the bending of the flat strip to a 0.125-in. radius.

As the drawing operation continues, the bent metal is unbent because the leg, in its final shape, is straight. During the unbending step, this same element of metal is again compressed to a strain of –0.014 and finally stretched to a strain of 0.014 for a net total strain of 0. The final thickness is 0.125-in. However, the metal has received a total of four cycles of strain, each equal to 0.014. The value of the equivalent strain is determined as follows:

$\varepsilon_{qus} = 0.014 + 0.014/2 + 0.014/3 + 0.014/4 = 0.029$

which is approximately 3% cold work.

Finally, the tensile yield strength at the midthickness in the 30-in. direction can be calculated as follows:

$(S_y)_{tBo} = 200[0.029/1 + 0.2 \times 0.029)^{0.30} = 69$ MPa

or nearly twice the original value.

As a final example, consider the experimental data shown in Fig. 1. The average monotonic tensile test data for the brass alloy 260 used in that project are proportional limit, 67 MPa; yield strength, 76 MPa; tensile strength, 303 MPa; strength coefficient σ, 758 MPa; m, 0.60; and fracture strain, 1.21. The compressive strain for 19% axial cold work is $E_w = 0.21$.

Based on the above original properties of alloy 260, the six yield strengths of the compressed block can be calculated as follows:

For specimen No. 7:

$(S_y)_{tLc} = 758(0.105/1.21)^{0.60} = 167$ MPa

For specimen No. 8:

$(S_y)_{cLc} = 758(0.21/1.042)^{0.60} = 291$ MPa

For specimens No. 9 and 11:

$(S_y)_{tTt} = 758(0.21/1.105)^{0.60} = 281$ MPa

For specimens No. 10 and 12:

$(S_y)_{cTt} = 0.90 \times 281 = 253$ MPa

The agreement between the calculated values and the experimentally determined values is very good. The larger discrepancies are with the compressive yield strengths, and this is mostly due to the difficulty in experimentally measuring the compressive yield strength.

These three examples illustrate the techniques of calculating the strength of the material in parts made by the cold working processes, including cyclic straining. These rules of strain strengthening have also been found to predict quite reliably the strength distribution in solid cylinders after cyclic torsional deformation as well as other forming operations.

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